

## Interest Rate Model “Financial Market” Calibration

Agronin, Borison, Hamm; October 18, 2015 **DRAFT**

### Introduction

Long-term interest rates over the next 5-10 years are critical to Manitoba Hydro’s financial performance. The future is uncertain, and Hydro’s goal is to reflect this uncertainty in the most accurate and credible interest rate forecast possible. Earlier, Hydro relied primarily on expert judgment and consensus to develop high, reference and low interest rate forecast trajectories. More recently, Hydro has used a publicly-available stochastic interest rate model developed by the American Academy of Actuaries and the Society of Actuaries.<sup>1</sup> This model is based primarily on historical U.S. interest rates, and produces a time series going forward that captures both longer-term multi-year trends and shorter-term intra-annual variation. This output can be used along with the output of other stochastic models, such as models of rainfall or commodity prices, to get a better picture of Hydro’s financial risk using its financial forecasting model FinFor.

This memorandum describes work by BRG to assist Hydro to take a step further in its treatment of interest rates, and ensure that the stochastic interest rate model is consistent with current prices of relevant financial market instruments. There is considerable research and numerous articles on the use of financial instruments, such as futures and options, in commodity price and interest rate forecasting. Broadly speaking, the predominant conclusion is that forecasting of these market variables is (and, in theory, should be<sup>2</sup>) difficult, and that financial instrument information – while imperfect - should be used in such forecasts either alone or in combination with other methods.<sup>3</sup> Overall, using available financial instruments such as futures and options in forecasting is considered “best practice.”

### Approach

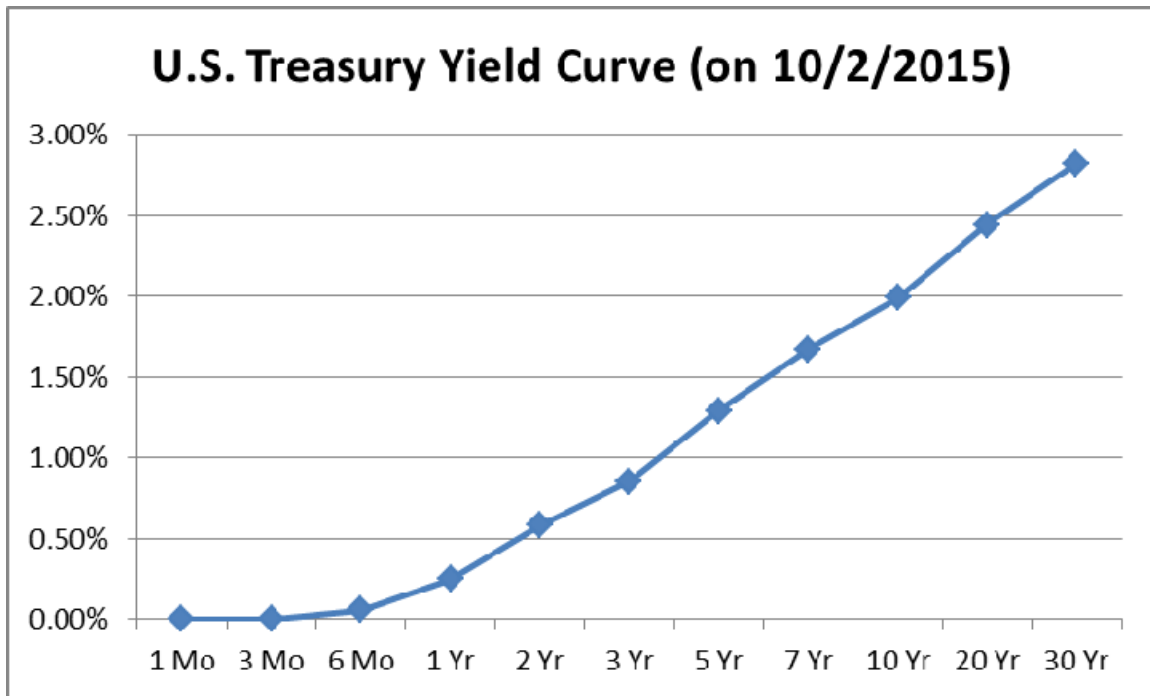
Interest rates at a point in time are typically represented by a “term structure” or “yield curve” that displays the interest rate for bonds of different durations (3-month, 1 year, 10 year, etc.). See current figure below.

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<sup>1</sup> The actuarial model is available at <https://www.soa.org/research/software-tools/research-scenario.aspx>.

<sup>2</sup> If it were easy, we’d all be trillionaires.

<sup>3</sup> See Chakriya Bowman and Aasim M. Husain, *Forecasting Commodity Prices: Futures vs. Judgment*, IMF Working Paper WP/04/41, March 2004 for a comparison of commodity price forecasts using judgment, historical data and futures. To quote, “futures-based models yield better forecasts than historical-data-based models or judgment, especially at longer horizons.” Consequently, these authors recommend using time-series models calibrated to futures data, similar to our suggested approach. See Riccardo Cesari and Lorenzo Sevini, *Using Options to Forecast LIBOR*, SSRN Electronic Journal, September 2004 for a comparison of interest rate forecasting methods. Their conclusion is that an options-based approach is superior to either consensus forecasts or futures alone. Our suggested approach incorporates both options and futures.



To represent future interest rates, one describes the evolution of the yield curve over time. This yield curve is typically measured with three factors - level, slope and curvature. The level factor, which reflects parallel shifts in the curve, is the most important and is typically regarded as explaining about 90% of the variation in interest rates.<sup>4</sup> Taken together, the three factors – level, slope, curvature - are typically regarded as explaining more than 95% of the variation in interest rates.<sup>5</sup>

Hydro uses its current stochastic model to forecast the entire term structure – level, slope and curvature. While a variety of financial market instruments associated with interest rates are available, they are really most useful for inferring the level of future interest rates, not the slope and curvature of the yield curve.<sup>6</sup> Research indicates that historical data can be quite useful in forecasting slope and curvature, but less so in forecasting level.<sup>7</sup>

Our approach is to make use of available data on current futures and options prices to calibrate the level of the interest rate yield curve emerging from the current stochastic model, and to rely on the current model (and its underlying historic data) to estimate the slope and curvature of the yield curve. The result is a calibrated stochastic model that combines available historical and current data to forecast the entire term structure.

<sup>4</sup> R. Litterman, and J. Scheinkman, *Common Factors Affecting Bond Returns*, Journal of Fixed Income, Volume 1, p. 58, 1991.. Excess bond returns are defined as bond returns in excess of the overnight repo rate.

<sup>5</sup> *Id.*

<sup>6</sup> Futures and options are useful for inferring level; forwards based on the yield curve are useful for inferring slope and curvature and, to a lesser extent, level.

<sup>7</sup> Gregory R. Duffee, *Forecasting Interest Rates*, Handbook of Economic Forecasting, Volume 2, Part 1, p. 386, July 2012.

## Forecasting Procedure and Results

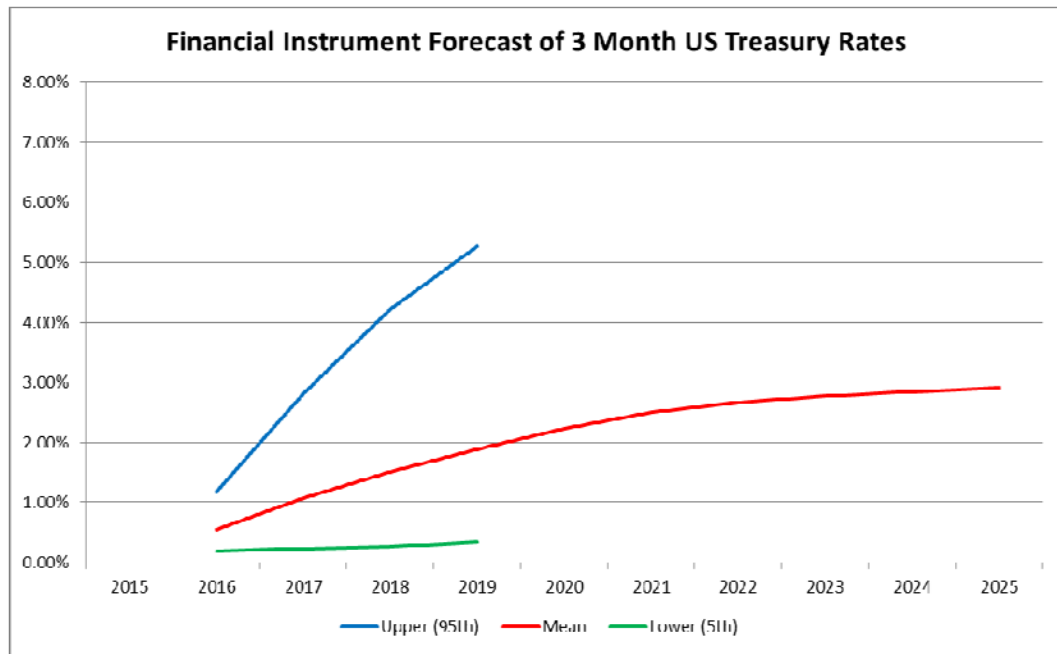
Our forecasting procedure and current results are described below.

1. Collect data on the price of Eurodollar futures and options, reflecting what market participants anticipate the 3-month USD LIBOR rate will be in the future. Eurodollar futures are quarterly and extend 10 years into the future (overall, 40 quarters); Eurodollar options are quarterly and extend 5 years into the future. Trillions of dollars of these futures and options are traded globally, making their prices effectively a “crowd consensus.” Of course, the liquidity in these markets declines substantially over the 5-year (options) or 10-year (futures) horizon. This Eurodollar information is readily available on Bloomberg. As shown in the figure below, it is also available on the CMEgroup website. See snapshot below of 10/2/2015 quotes for futures contracts. Similar information is available for options.

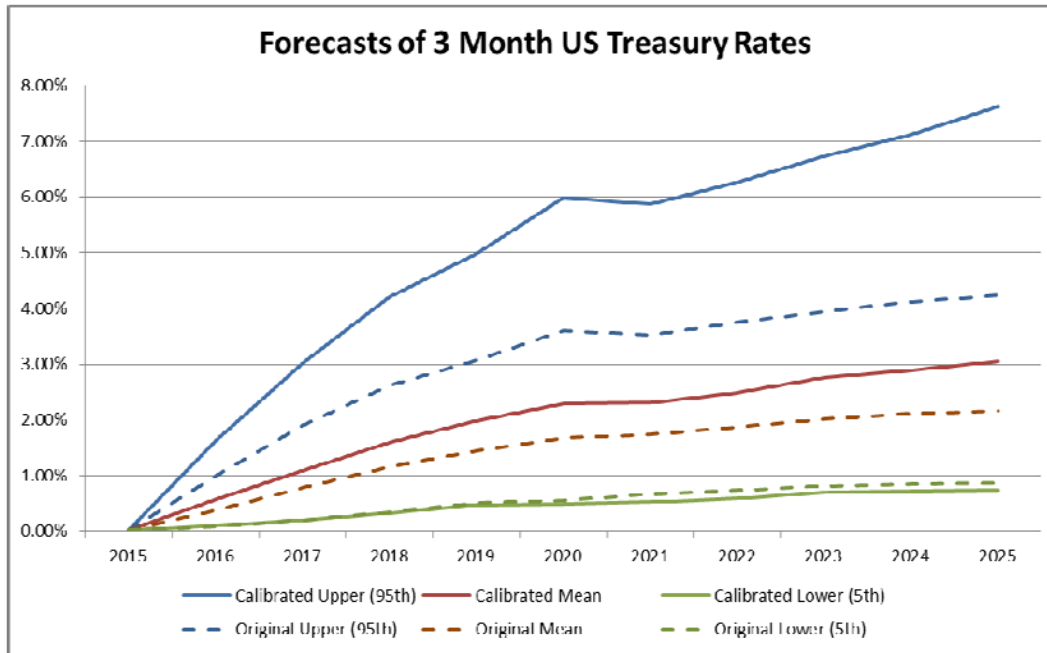
Month	Options	Charts	Last	Change	Prior Settle	Open	High	Low	Volume	Updated
OCT 2015	<a href="#">OPT</a>	<a href="#">Charts</a>	99.6825	+0.0025	99.68	99.6675	99.6825 b	99.6625	23,906	17:21:57 CT 02 Oct 2015
NOV 2015	<a href="#">OPT</a>	<a href="#">Charts</a>	99.675	+0.015	99.66	99.630	99.675 b	99.620	20,142	17:21:57 CT 02 Oct 2015
DEC 2015	<a href="#">OPT</a>	<a href="#">Charts</a>	99.615	-0.005	99.62	99.580	99.640	99.570	453,278	17:21:57 CT 02 Oct 2015
JAN 2016	<a href="#">OPT</a>	<a href="#">Charts</a>	99.590	+0.01	99.58	99.530	99.590 b	99.525	829	17:21:57 CT 02 Oct 2015
FEB 2016	<a href="#">OPT</a>	<a href="#">Charts</a>	99.575	+0.025	99.55	99.570	99.575 b	99.570	100	17:21:57 CT 02 Oct 2015
MAR 2016	<a href="#">OPT</a>	<a href="#">Charts</a>	99.515	-0.005	99.52	99.460	99.555	99.455	431,407	17:21:57 CT 02 Oct 2015

2. Use the Eurodollar futures and options prices to develop a forecast of USD LIBOR interest rates, and convert this forecast to a forecast of 3-month US T-bill rates using the historical TED spread. The futures provide a mean forecast for ten years, the options provide an upper (95<sup>th</sup> percentile) and lower (5<sup>th</sup> percentile) forecast for five years. See current forecast extending 10 years below. As the forecast indicates, market expectations are that 3-month rates will rise from their current level of below 1% to between 1% and 2% within 3-4 years and to nearly 3% within 6-8 years. There is

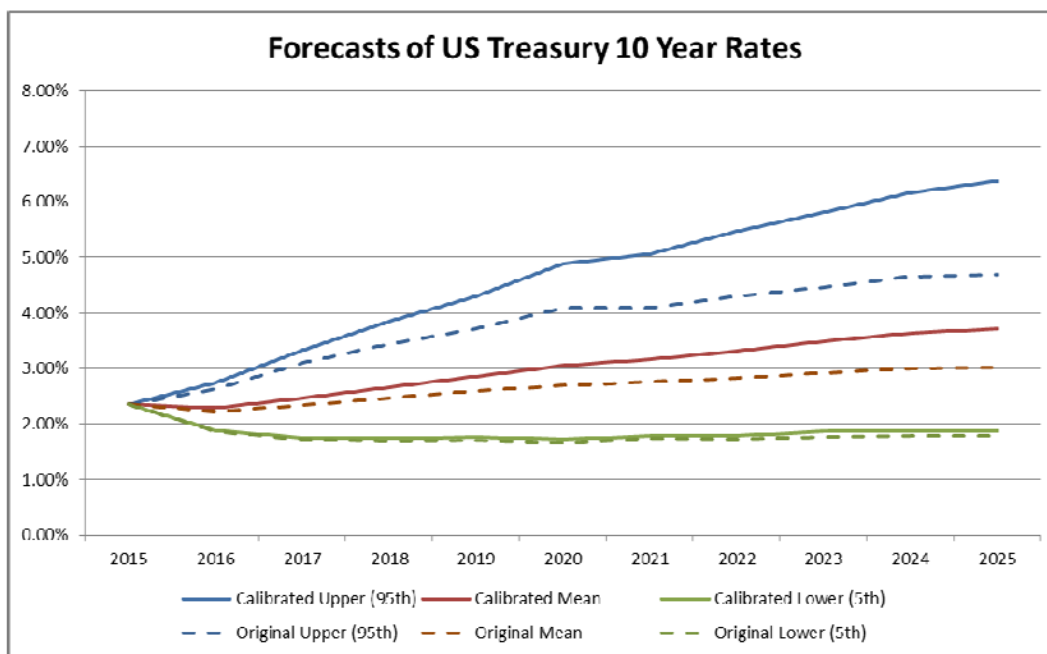
considerable uncertainty around these expectations with the 90% confidence band ranging from essentially 0% to over 5% within five years.



- Calibrate two key parameters of the actuarial model so that the forecast of 3-month US T-bill rates matches the mean and spread of the financial forecast. The parameters have been fitted to the mean and standard deviation of the financial forecast by minimizing the mean square error, which is the square of the difference between the means and standard deviations of forecasts generated by the actuarial model and prices of interest rate futures. See below for the current comparison of the original model forecast using the actuarial model and the calibrated model forecast extending 10 years. As the figure indicates, the calibration raises the original mean forecast moderately – by 0.50 to 0.75%. It raises the original upper forecast significantly – by 2.5 to 3.5%. The original lower forecast is essentially unchanged.



4. Use the calibrated model to develop forecasts of other US interest rates, such as the 10-year bond. As noted earlier, this reflects the original model treatment of yield curve slope and curvature. See current original and calibrated 10-year bond forecasts below extending 10 years. As the figure indicates, the calibration raises the original mean forecast slightly – by 0.25% to 0.75% - and the original upper forecast moderately – by 0.75% to 1.5%. The original lower forecast is essentially unchanged.



5. Convert the model-generated USD interest rate forecasts into CAD interest rate forecasts using the following formula:

$$i_{CAD}^m = \frac{(1 + i_{CAD}^f)}{(1 + i_{USD}^f)} (1 + i_{USD}^m) - 1$$

where  $i$  refers to interest rate, superscript “f” refers to “forward” and superscript “m” refers to “model.” The forward CAD and USD interest rates are obtained through interpolation and extrapolation of the available US and Canadian yield curves. Linear interpolation is used for bonds lying within a 30-year window (e.g., a 10 year bond in 10 years or a 1-year bond in 20 years). Extrapolation based on the Nelson-Siegel method (the same term structure as in the actuarial model) is used for bonds extending beyond the 30-year window (e.g., a 10-year bond in 30 years or a 30-year bond in 10 years). Extrapolation is necessary because the current US and Canadian term structure “only” extends 30 years. The USD model interest rates are obtained directly from the calibrated model. This formula is based on three key assumptions.

- Interest rate parity<sup>8</sup> as follows:

$$i_{CAD} = (1 + i_{USD}) \frac{F_{CAD/USD}}{S_{CAD/USD}} - 1$$

Interest rate parity is fundamental to modern finance. It is discussed in most basic corporate finance<sup>9</sup> and international finance<sup>10</sup> textbooks, and widely discussed in the economic/finance literature.<sup>11</sup>

- Identical US and Canadian bond risk premia<sup>12</sup> or  $\rho$  in the following formula:

$$(1 + i^f) = (1 + i^e)(1 + \rho)$$

This assumption is also commonly made for countries where there are minimal barriers to the cross-border flow of capital, such as Canada and the U.S.<sup>13</sup>

- Hedged exchange rate risk. This approach incorporates the role that expected US-Canadian exchange rates play in interest rates, but not the added interest rate risk associated with

<sup>8</sup> See, for example, G. Caprio, The Evidence and Impact of Financial Globalization, Academic Press, 2012.

<sup>9</sup> See, for example, Richard Brealey and Stewart Myers, Principles of Corporate Finance, 6<sup>th</sup> Edition, 2000, p. 797 and Stephen Ross et. al., Corporate Finance, 8<sup>th</sup> Edition, 2008, p. 880.

<sup>10</sup> See, for example, Maurice Levi, International Finance, 5<sup>th</sup> Edition, 2009, pp. 113-142.

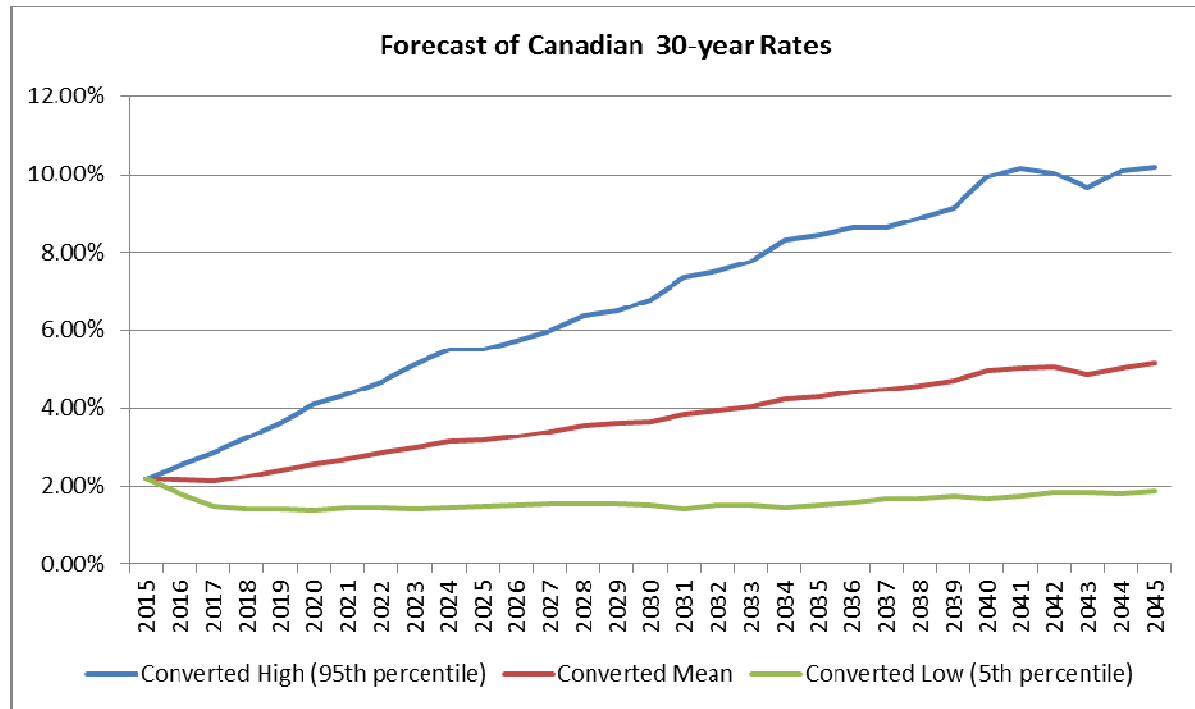
<sup>11</sup> See, for example, Richard Levich, Evidence on Financial Globalization and Crises: Interest Rate Parity, NYU Working Paper 2451/29949, September 2011.

<sup>12</sup> See, for example, K. Redhead, Personal Finance and Investments: A Behavioural Finance Perspective, Routledge, 2008.

<sup>13</sup> See, for example, Stephen Ross et. al., Corporate Finance, 8<sup>th</sup> Edition, 2008, p. 882.

exchange rate uncertainty. This is consistent with Hydro's strategy of hedging exchange rate risk.

See current CAD 30-year bond forecast below extending 30 years.



### Additional Analysis

As noted earlier, long-term interest rates are critical to Hydro's financial future. Consequently, it is important that interest rate forecasts be as accurate and credible as possible. The core approach reflected in this memo – using a well-established underlying stochastic interest rate model and developing parameters that are based appropriately on the latest historical data, financial instrument prices and expert judgment – is both sound and state-of-the-art. Two forms of additional analysis in the near term may be particularly helpful in enhancing this core approach.

Improved calibration. Accuracy, as well as credibility, can be improved by 1) using forwards in addition to futures and options, 2) using historical data on futures and options prices in addition to current data, and 3) using maximum likelihood rather than mean square error for fitting parameters.

Further backtesting. Credibility, as well as accuracy, can be improved by further backtesting of the calibrated model. Specifically, the model can be applied at past points in time (say in 2010) and model results compared with actual interest rate outcomes (say in 2015). This will help confirm the quality of the forecasts. In addition, these results can be compared to results using other approaches - the

original actuarial model and consensus forecasts. This will help confirm that the calibrated model is an improvement over previous approaches.